PINKNEY

An Absolute Determination of the Acceleration of Gravity

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AN ABSOLUTE DETERMINATION OF THE ACCELERATION OF GRAVITY

BY

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THESIS

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I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

LESLIE ARTHUR PINKNEY

ENTITLED AN ABSOLUTE DETERMINATION OF THE ACCELERATION OF GRAVITY

BE ACCEPTED AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF MASTER OF ARTS IN PHYSICS

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I. HISTORICAL

For the absolute determination of the acceleration of gravity ("g"), the compound reversible pendulum is usually employed. This was first used by Captain Kater in 1817, who was commissioned by the Royal Society of London to determine accurately the length of a seconds pendulum at Greenwich.[1] In the construction of the pendulum, he made the first practical application of the theory of reversibility of the centers of suspension and oscillation that had been worked out in 1673 by Huyghens. By determining the period with the method of coincidences, which was used first by Bouguer in 1737, Kater succeeded in establishing a standard method for determining "g". Later in 1826, Bessel,[2] using a wire suspended pendulum, made extensive experiments in which he considerably modified the air corrections.

In this country the most extensive work done along this line was carried on under the supervision of the United States Coast and Geodetic Survey. The work of the Survey began about 1873 and extended over a number of years. Gravity determinations were made at several stations thruout the country. These were only differential or relative determinations based on the absolute value at Washington the base station. This absolute value was obtained from several independent determinations, one by Peirce at Hoboken using the Repsold reversible pendulum, and another by Her-

1. Poynting and Thomson,"Properties of Matter", p. 12
   Phil. Trans. for 1818 and 1819.
2. Ostwald,"Klass. der Exak. Wissen.", Vol. 7, 1889.

schel at Washington using the Kater reversible pendulum.[1] The mean of these results gave for the absolute value at Washington 980.10 cm/sec^2., which was used as the basis for the relative work at the other stations.

The purpose of this work is to determine the absolute value of the acceleration of gravity in room 116 on the first floor of the Laboratory of Physics at the University of Illinois. A Kater's pendulum is swung in a receiver from which the air is exhausted. In this way the period of the pendulum may be obtained independent of the effects of the air, thus giving an absolute value of "g".

1.United States Coast and Geodetic Survey Report,1891,Part II.

II. THEORETICAL

Kater's compound reversible pendulum is based directly on Huyghens' principle that the centers of oscillation and suspension are interchangeable, and that the distance between the two is equal to the length of an equivalent simple pendulum. A brief theoretical treatment of this will now be considered.

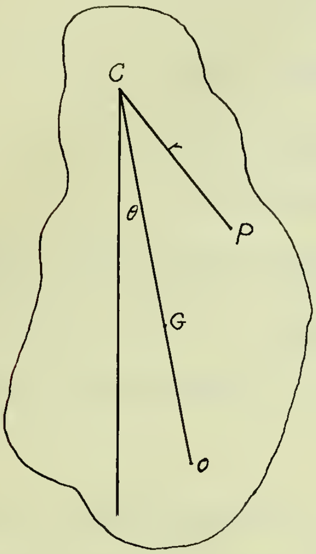

Figure 1

Let any irregular body be capable of rotation about an axis passing thru the point C, the center of suspension. Let w be the angular velocity of any little particle P at a distance r from the axis C. Then

$$wr = \text{linear velocity of P.}$$

Since acceleration is the derivative of the velocity with respect to the time,

$$r\,\frac{dw}{dt} = \text{linear acceleration of P.}$$

If m is the mass of the particle P, then the force acting on P at right angles to r is, since F = ma,

$$F = mr\,\frac{dw}{dt}$$

The moment of this force about the axis thru C is

$$mr^2\,\frac{dw}{dt}$$

The total moment of the force for the whole body is the summation of the moments for all the little particles.

$$\text{Total moment} = \frac{dw}{dt}\Sigma mr^2$$

Now let M = total mass of the body

G = center of gravity

h_1 = distance CG

θ = angle of displacement from a vertical plane thru (C

g = acceleration of gravity.

Since the total weight of the body may be considered as concentrated at G, the moment of the force of gravity about the vertical plane thru C is

$$Mgh_1 \sin\theta.$$

For a condition of equilibrium the moment of the force of gravity must be equal to the moment of the force tending to produce rotation. The moment of the force of gravity may be considered negative since it is in opposition to the moment of the rotational force. Therefore

$$\frac{dw}{dt}\Sigma mr^2 = -Mgh_1 \sin\theta.$$

But $$w = \frac{d\theta}{dt}$$

Then $$\frac{d^2\theta}{dt^2}\Sigma mr^2 = -Mgh_1 \sin\theta.$$

Now if the body is allowed to swing thru very small angles only, the sine may be replaced by the angle itself, when expressed in radians. Then,

$$\frac{d^2\theta}{dt^2} = - \frac{Mgh_1}{\Sigma mr^2}\ \theta. \qquad (1)$$

This means that the acceleration is proportional to the displacement and the point P will describe a simple harmonic motion.

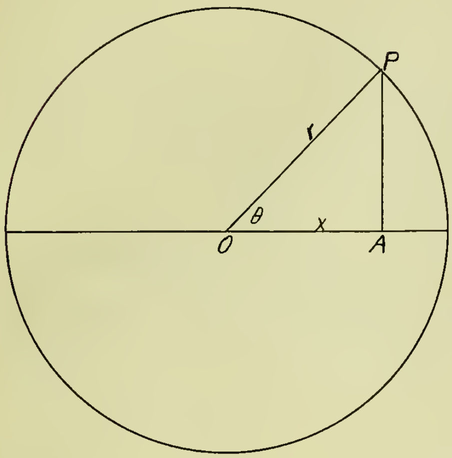

Figure 2

Now let the point P be considered as moving in the circle of reference which is the projection of the simple harmonic motion. (Fig. 2) Its acceleration toward the center is given by the usual formula,

$$a = \frac{v^2}{r}$$

But the desired acceleration is that toward the center along AO = x. Resolving the acceleration along PO,

$$a_x = - \frac{v^2}{r} \cos\ \theta = - \frac{v^2 x}{r^2}$$

But $$v = wr$$

Then $$a_x = - \frac{w^2 r^2 x}{r^2} = -w^2 x.$$

Also $$w = \frac{2\pi}{T}$$

where T is the periodic time. Therefore

$$a_x = - \frac{4\pi^2}{T^2} x,$$

that is the acceleration toward the center is proportional to the displacement. This is the corresponding condition for the compound pendulum that is stated in equation (1). Then the

constants must be equal, or

$$-\frac{4\pi^2}{T^2} = -\frac{Mgh_1}{\Sigma mr^2}$$

From this the period of vibration of the pendulum about C is

$$T_1 = 2\pi\frac{\sqrt{I}}{\sqrt{Mgh_1}} \qquad (2)$$

where $\Sigma mr^2 = I$, which is the moment of inertia.

Now let the body be rotated about the center of oscillation O (Fig. 1) distant h_2 from G and in the same plane as C and G. In the same manner as above the period may be shown to be,

$$T_2 = 2\pi\frac{\sqrt{I}}{\sqrt{Mgh_2}} \qquad (3)$$

Let I_o be the moment of inertia of the body about its center of gravity. Then by a well known principle in Mechanics, the moment of inertia about the point C is,

$$I = I_o + Mh_1^2$$

Substituting this in equation (2),

$$T_1 = 2\pi\,\frac{\sqrt{(I_o + Mh_1^2)}}{\sqrt{Mgh_1}} = 2\pi\,\frac{\sqrt{l}}{\sqrt{g}}$$

for there is some simple pendulum of length l whose period is the same as that of the compound pendulum under consideration. Therefore,

$$\frac{I_o + Mh^2{}_1}{Mh_1} = l$$

In this equation I_o may be replaced by MR^2, where R is the radius of gyration. Then,

$$l = \frac{R^2 + h_1^2}{h_1} = \frac{R^2}{h_1} + h_1 \qquad (4)$$

Now let O (fig. 1) be placed so that CO = l. Then when the body is suspended from O, the time of vibration from equation (3) is,

$$T_2 = 2\pi \frac{\sqrt{I}}{\sqrt{Mgh_2}} = 2\pi \frac{\sqrt{(MR^2 + Mh_2^2)}}{\sqrt{Mgh_2}} = 2\pi \frac{\sqrt{l'}}{\sqrt{g}}$$

whence l' may be obtained, which is the equivalent length of a simple pendulum when suspended from O.

$$l' = \frac{R^2 + h_2^2}{h_2} = \frac{R^2}{h_2} + h_2 \qquad (5)$$

But , from equation (4),

$$\frac{R^2}{h_1} = l - h_1 = h_2$$

Substituting these values in (5),

$$l' = \frac{R^2}{R^2/h_1} + l - h_1$$

or, $$l' = l.$$

That is , the equivalent length of a simple pendulum is the same whether the body is suspended from C or O. Hence, the center of suspension and the center of oscillation are interchangeable and the distance between the two may be taken as the equivalent length of a simple pendulum.

In this work the usual form of Kater's pendulum was used, in which the knife-edges are fixed. An adjustable weight is provided so that the periods about the knife-edges may be made very nearly equal. In order to use the simple pendulum equation for the calculation of "g", the periods in the two cases

would have to be exactly equal. The adjustment to equality is a task requiring some patience and time. Instead, Bessel introduced the following modification so that it is necessary for the periods to be only approximately equal.

From equation (2), remembering that $I = MR^2 + Mh_1^2$, and taking the period for a single vibration,

$$t_1 = \pi \frac{\sqrt{(R^2 + h_1^2)}}{\sqrt{gh_1}}$$

In the same way from equation (3),

$$t_2 = \pi \frac{\sqrt{(R^2 + h_2^2)}}{\sqrt{gh_2}}$$

Eliminating R^2 from these two equations, the final formula for the calculation of "g" is obtained.

$$\frac{\pi^2}{g} = \frac{h_1 t_1^2 - h_2 t_2^2}{h_1^2 - h_2^2}$$

$$= \frac{t_1^2 + t_2^2}{2(h_1 + h_2)} + \frac{t_1^2 - t_2^2}{2(h_1 - h_2)} \qquad (6)$$

Since the periods are very nearly equal, the difference of their squares is very small, so that the value of the second fraction in (6) is very small. Hence it is necessary to know $(h_1 - h_2)$ only approximately. The position of the center of gravity is readily determined by balancing the pendulum, then h_1 and h_2 can be measured with a meter stick. The quantities that must be measured with great precision are $(h_1 + h_2) = l$ and t_1 and t_2.

III. DESCRIPTION OF APPARATUS

A Kater's pendulum, as shown in Plate I (a), was used. It consists of a long strip of brass 1 1/2" wide and 1/8" thick and 76" long. Steel knife-edges are placed at equal distances from the ends so that the distance l is a little more than that of a seconds pendulum . Between one knife-edge and the nearer end is placed a fixed weight of brass, 3 1/2" in diameter. Two other adjustable weights are placed between the knife-edges, one at A for rough adjustments and another at B, which is fitted with a tangent screw for finer adjustments. The two knife-edges face each other so that the pendulum may be conveniently suspended from either.

This pendulum was swung in a specially constructed air-tight iron receiver, shown in Plate I (b). An iron pipe 6" in diameterand 39" long is fitted at each end with a cast iron collar so that a larger pipe 10" in diameter and 11" long can be securely attached. Another collar is necessary at both the top and bottom in order to allow another 6" pipe to be attached to accommodate the ends of the pendulum. This cap may be screwed on permanently at the bottom, but the top cap must be arranged so that it can be removed when it is desired to reverse the pendulum. To do this the cap is fitted on opposite sides with slots which fit into short bolts that are imbedded in the collar . To insure a good joint a rubber gasket

PL 2340, made by Max Kohl.

is placed over the bearing surface on the collar. Then to remove the cap it is only necessary to take off the nuts at nn. The knife-edges are supported by an 8" plate in which is cut a 5/8" slot as shown in (c). The position of this plate in the receiver is shown by the dotted lines. The plate is held in place by a small pin which readily slips thru the hole at e into another hole drilled for the purpose in the collar. For observing the swinging pendulum, four windows are provided, two in front as shown in the figure and two in the rear. To provide a plane bearing surface for the glass, brass strips are cut and soldered to the rounding surface of the pipe. In order to withstand the pressure, the pieces of glass used were one inch in thickness. The glass is held to the bearing surface by clamps which are tightened by nuts which turn on small bolts imbedded in the pipe. By turning the plate already referred to thru 90°, it is possible to have the pendulum swing in a plane parallel or perpendicular to the plane of the windows. To set the pendulum swinging, after the air has been pumped out, it is held aside by a small fuse wire attached to two binding posts bb, which are carefully insulated from the receiver. The pendulum may then be started at any time by blowing the fuse. The air is exhausted with an air pump[1] thru a stopcock S. The pressure is measured by attaching a closed short-arm manometer filled with mercury to another stopcock at S'.

The firm support of this receiver is a matter of considerable importance. A plank seven feet long was fastened by

1. PL 3576, Gaede rotary pump, E. Leybold's Nachfolger.

long expansion bolts to a solid brick wall. To the plank was bolted a heavy cast iron wall-bracket. The receiver is then suspended from this bracket, the ends of which are shown at cc. In this way a very rigid support was provided for the swinging pendulum. For convenience a much lighter support was provided for swinging the pendulum in the open air. This was done so that the periods about the two knife-edges could be readily adjusted to approximate equality before the pendulum was placed in the receiver.

In measuring the period, the optical coincidence method, to be described later, was employed. This requires two mirrors, one fixed and the other moving with the pendulum. The movable mirrors are stuck to the pendulum with a small piece of wax at m and m'. When the pendulum is swinging with the weight down m is used for observation, and when the weight is up m' is used. The fixed mirror must be supported very near to the moving one and yet not interfere with the motion of the pendulum. It must also be arranged so that, when the pendulum is at rest, the two images of a spark may be adjusted to exact coincidence in the field of an observing telescope. To do this the support shown in (d) is employed. This is constructed so that it will rest conveniently at one side of the pendulum on the shelf formed by the collar in the lower part of the receiver. It consists of an iron base to one end of which is attached a short upright piece. To this piece is fastened an iron strip carrying a thumbscrew s. A second and thinner strip is fastened just above so that one end rests on the thumbscrew and the other is screwed to the top of the upright piece. The mirror is then waxed to another upright piece at p,

which projects out over the base as shown in the end view. By turning the thumbscrew, the mirror may be adjusted vertically, and by moving the whole support , it may be adjusted horizontally. In this way the two images may be brought to exact coincidence when the pendulum is at rest.

In order to keep the receiver from leaking, it was found necessary to go over all the joints with half-and-half wax. This is a mixture of half resin and half beeswax melted together and is applied with a brush when hot. The windows were seated in universal wax. This was found more convenient because the lower window must be removed each time a series of coincidences is observed. The whole surface of the receiver was treated with a special iron filler and then painted . With these precautions, the pressure could be kept constant at 1.5 cm. When the pump was turned off, the leakage was so small that the pressure remained the same for several hours.

After completing the work, it was found that several alterations in the receiver could be profitably made. The two rear windows are unnecessary. Another smaller window should be provided 90° around to the side of the lower front window. This should be so placed that the lower knife-edge may be readily observed thru it. In this way the amplitude of vibration may be measured accurately with an observing telescope on a scale. The distance between the knife-edges may be taken as the distance to the scale, thus allowing an accurate calculation of the correction for arc.

In the plate supporting the knife-edges should be

placed an insert of hardened steel or preferably agate. It was found that the knife-edges made a slight impression in the iron thus considerably increasing the friction of support.

The binding posts for the fuse wire were found to be unnecessary. It was possible to set the pendulum swinging with the hand, seat the window in the wax and exhaust the receiver to a constant pressure in about twenty minutes. Ample time remains to take the observations since the pendulum continues to swing thru a sufficient amplitude for about three hours.

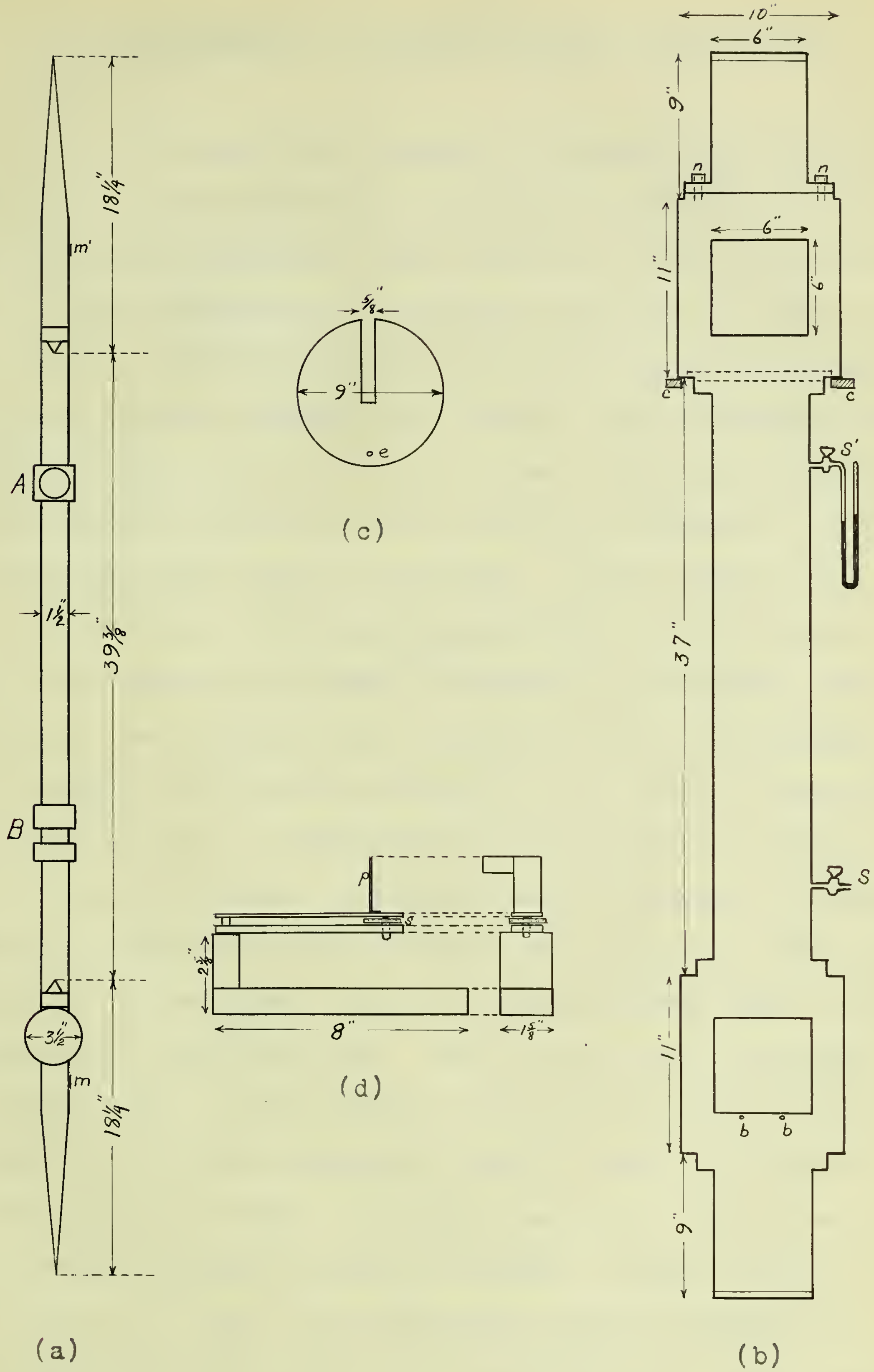
18¼"
m'
A
1½"
39⅜"
B
3½"
m
18¼"
(a)
5/8"
9"
e
(c)
p
S
b
2⅝"
8"
1⅝"
(d)
10"
6"
9"
n
n
11"
6"
6"
c
c
S'
37"
S
11"
b
b
9"
(b)

PLATE I.

IV. MEASUREMENT OF DISTANCE BETWEEN KNIFE-EDGES

To measure the length l, a cathetometer resting on a heavy pier and provided with two reading telescopes was employed. The pendulum was suspended from a supporting frame immediately in front of the cathetometer. A standard meter bar of invar, correct at 0° C., was supported alongside the pendulum, being adjusted to a vertical position by means of a level. The cathetometer was adjusted so that the optical axis of each telescope was parallel with the axis of its level, and so that, by turning in a horizontal plane the level was not disturbed. In the field of each telescope, in addition to the usual cross-hairs, is a horizontal hair which can be moved up and down by turning a micrometer screw. By sighting on the standard bar, a constant K can be determined for each telescope, which may be defined as the number of centimeters thru which the hair is advanced when the micrometer screw is turned thru one division. To determine K an average of several trials was taken as the final value. The telescopes were then sighted on the pendulum, each hair being adjusted until it just coincided with the knife-edge. Then, turning the telescopes on the standard bar, it was observed thru how many divisions it was necessary to turn the micrometer screw in order to bring the hair to a division on the bar. By making use of K, the distance between the knife-edges is readily obtained. Since the bar is a meter long at 0° only, the temperature must be observed and a correction applied making use of the coefficient of linear expansion of invar. The accompanying table shows the results of the measurement.

TABLE I.

Data for the measurement of the distance between the knife-edges of the pendulum by means of the cathetometer.[1]

Trial	Div. from knife-edge to 0.0 of scale Upper	Lower	Corresponding distance in cm. Upper	Lower	Distance bet. knife-edges
1	25	110	.0090	.0407	99.9503
2	50	87	.o180	.0322	99.9498
3	62	75	.0223	.0277	99.9499
4	57	79	.0205	.0292	99.9503
5	59	78	.0212	.0288	99.9499
		Mean apparent length,			99.9500

Temperature	24°C
Linear coefficient of invar[2]	$.87 \times 10^{-6}$
Expansion from 0°C to 24°C	.0021 cm.
Corrected length at 24°C	99.9521 cm.
K for upper telescope	.00036 cm.
K for lower telescope	.00037 cm.

1. PL 3271, Geneva Society, Switzerland.
2. Watson, "Text-book of Physics" p. 208.

[illegible]

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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[illegible]		[illegible]	[illegible]	

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V. MEASUREMENT OF THE PERIOD

The measurement of the period must be taken with extreme care, since any error made in this enters as the squares into the calculation of "g". As already described, the pendulum is swung in a receiver from which the air is exhausted to a pressure of 1.5 cm. of mercury. The period of the pendulum while swinging in the receiver was compared to that of a standard astronomical clock made by Riefler of Munich.

The method used was the optical method of coincidences. This is essentially the method used by the United States Coast and Geodetic Survey but adapted to the conditions of this work. Within the works of the standard clock is a device which automatically breaks an electrical circuit each second. This circuit is connected thru a relay to the primary of an induction coil. By short-circuiting the automatic make-and-break of the coil, the relay is made to break the primary circuit. Thus the clock causes a spark to pass between the secondary terminals once each second.

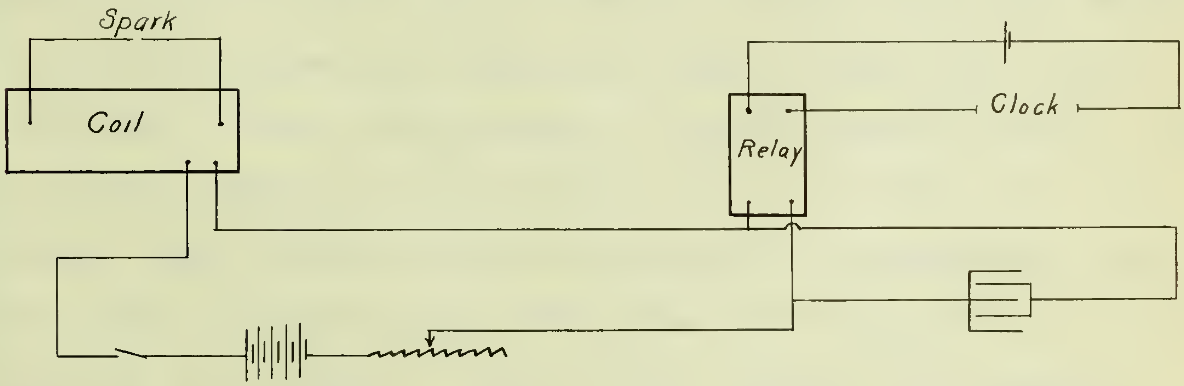

Figure 3

Figure 3 shows the method of connection. One storage cell is

used in the clock circuit and five or six with a rheostat are used in the relay circuit. In order to reduce the amount of sparking of the relay to a minimum a condenser is connected as shown.

The image of the spark that is thus produced is reflected into the field of an observing telescope by two mirrors, one fixed and the other movable. The movable mirror is placed on the pendulum and the fixed mirror is firmly supported as near to the movable one as possible without interfering with the motion of the pendulum. These two mirrors are so adjusted that, when the pendulum is hanging freely at rest, the two images of the spark appear as one in the field of the telescope. When the pendulum is set swinging the images are separated and gradually become farther and farther apart as the pendulum loses on the clock. The movable image finally goes out of the field altogether but soon reappears as the next coincidence is approached. At the instant the spark passes and only one image is seen, the hour, minute and second as noted on the clock are recorded. The time of the first three and last three of a series of twelve coincidences is noted. Then the time for any desired interval is calculated, for instance the interval between the first and tenth or second and twelfth coincidence. Since the pendulum is slightly slower than the clock, one single vibration is lost during the time between two coincidences. Thus between the second and twelfth coincidences the pendulum makes ten less single vibrations than the number of seconds indicated by the clock. From this the period of the pendulum is readily obtained. With careful observation this method gives an accuracy of more than one part in a million.

The following tables show the results of the observations. Tables II and III give the data for determining t_1 and t_2. For purposes of comparison, table IV is given to show the results obtained when the pendulum is swinging at atmospheric pressure

TABLE II

Determination of t_1. Pendulum swinging with weight down.

Trial	No.of coin.	Time h.	m.	s.	Int-erval	No.of sec.	Period	Corr. for arc	Corr. period
1	1	8	25	18	1-12	4003	1.00275		
	2	8	31	22	1-11	3639	5		
	3	8	37	26	2-12	3639	5		
	10	9	19	53	1-10	3275	5		
	11	9	25	57	2-11	3275	5		
	12	9	32	1	3-12	3275	5		
					Mean,		1.00275	.00002	1.00273
2	1	10	26	53	1-12	3995	1.00276		
	2	10	32	57	1-11	3632	6		
	3	10	39	00	2-12	3632	6		
	10	11	21	22	1-10	3269	6		
	11	11	27	25	2-11	3268	6		
	12	11	33	28	3-12	3268	6		
					Mean,		1.00276	.00003	1.00273
3	1	1	23	9	1-12	4016	1.00274		
	2	1	29	13	1-11	3651	4		
	3	1	35	17	2-12	3652	4		
	10	2	17	55	1-10	3286	4		
	11	2	24	00	2-11	3287	4		
	12	2	30	5	3-12	3288	4		
					Mean,		1.00274	.00001	1.00273

Temperature 26.5°C. Mean corrected period, 1.00273

TABLE III

Determination of t_2. Pendulum swinging with weight up.

Trial	No.of coin.	Time h. m. s.	Interval	No.of sec.	Period	Corr. for arc	Corr. period
1	1	1 56 22	1-12	4302	1.00256		
	2	2 2 53	1-11	3910	6		
	3	2 9 24	2-12	3911	6		
	10	2 55 00	1-10	3518	6		
	11	3 1 32	2-11	3519	6		
	12	3 8 4	3-12	3520	6		
			Mean,		1.00256	.00003	1.00253
2	1	3 36 35	1-12	4312	1.00255		
	2	3 43 5	1-11	3920	5		
	3	3 49 36	2-12	3922	5		
	10	4 35 21	1-10	3526	5		
	11	4 41 55	2-11	3530	5		
	12	4 48 27	3-12	3531	5		
			Mean,		1.00255	.00002	1.00253
3	1	7 27 49	1-12	4297	1.00256		
	2	7 34 18	1-11	3905	6		
	3	7 40 48	2-12	3908	6		
	10	8 26 23	1-10	3514	6		
	11	8 32 54	2-11	3516	6		
	12	8 39 26	3-12	3518	6		
			Mean,		1.00256	.00003	1.00253
			Mean corrected period,				1.00253

Temperature 26.5°C.

TABLE IV

Determination of "g" at atmospheric pressure. No correction is made for arc. Pendulum swinging with weight down.

No.of coin.	Time h. m. s.	Interval	No.of sec.	Period
1	2 30 3			
2	2 35 16			
3	2 40 29			
10	3 17 9	1-10	3226	1.00319
11	3 22 21	2-11	3225	9
12	3 27 35	3-12	3226	9
		Mean period t_1		1.00319

Pendulum swinging with weight up.

No.of coin.	Time h. m. s.	Interval	No.of sec.	Period
1	10 15 5			
2	10 20 38			
3	10 26 10			
4	10 31 44			
16	11 33 2	1-16	4677	1.00321
17	11 38 40	2-17	4682	1
18	11 44 16	3-18	4686	1
19	11 49 49	4-19	4685	1
		Mean period t_2		1.00321

Temperature 23°C. h_1 = 57.1 cm. h_2 = 42.8 cm.

Value of "g" calculated from equation (6), 980.030

[illegible]

VI. CORRECTIONS

1. Temperature. The work of the experiment was carried on in an "even-temperature room". It was found that during the taking of the observations the temperature did not vary more than .2°, which is so small as to introduce a negligible error. The temperature however was not the same as that of the room in which the cathetometer measurement of the length was made. This of course necessitates a correction, which is readily made by noting the difference in temperature and making use of the linear coefficient of brass.

2. Pressure. By keeping the air pump in operation during the progress of an observation, the pressure in the receiver was kept down to a constant value of 1.5 cm. The period at this pressure will differ but very little from the period in a perfect vacuum[1], so that no correction is made for pressure.

3. Arc correction. The correction to infinitely small arcs is of considerable importance. To make this it is necessary to know the amount of displacement from the center in minutes of arc both at the beginning and end of a series of observations. A short centimeter scale is placed across the shelf in the enlargement of the lower part of the receiver. Its distance from the upper knife-edge is readily determined by adding to the length 1, the distance from the lower knife-edge to the surface of the scale. The position of the pendulum may be observed when at rest, when drawn aside at the beginning and again at the end of a series of

1. See page 26 of this thesis.

coincidences. In this way the number of centimeters of displacement is readily found. To determine the angle of displacement,

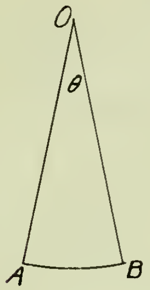

Figure 4

let OA = D, the distance from the knife-edge to the scale, and AB = a, the amplitude as measured on the scale. Then when θ is expressed in radians,

$$\theta = \frac{a}{D}$$

But there are 3438 minutes in one radian, so that in minutes of arc,

$$\theta = 3438 \frac{a}{D}$$

From this relation both the initial and final amplitudes are readily determined. The correction in seconds is taken from tables[1]. Since an increase in amplitude increases the period, this correction must be subtracted from the observed period. The application of the correction is made in tables II and III.

4. Rate of the clock. The clock employed for determining the period ticks seconds of mean solar time . It is electrically wound every 36 seconds and is kept in an"even-temperature" room so that atmospheric changes do not appreciably affect it. The rate as determined by the astronomical department was found to be .6 of a second fast per day. This will not affect the period within the sixth decimal place, so that this correction is not considered in the calculation.

1. Watson, "Practical Physics", p. 602.

VII. CALCULATION OF "g"

(a). From data obtained.

Mean corrected period t_1,	1.00273
Mean corrected period t_2,	1.00253
Length l in cm. at 24°C,	99.9521
Temperature of room during observations,	26.5°C
Linear coefficient of brass,	$.187 \times 10^{-4}$
Length l at 26.5°C,	99.9567
Distance h_1 in cm.,	57.1
Distance h_2 in cm.,	42.8
Value of "g" in cm./sec.^{2} from equation (6),	980.0993

(b). From Smithsonian Physical Tables.[1]

Formula given,

$$g_\phi = g_{45}(1 - .002662 \cos 2\phi)\left[1 - \frac{2h}{R}\left(1 - \frac{3}{4}\right)\right]$$

where ϕ is the latitude, h the elevation above sea level, R the radius of the earth, the surface density, the mean density of the earth. The ratio - is very nearly 1/2. This gives a diminution in "g" Of .00588 cm./sec.^{2} for each 100 feet of elevation.

Elevation of laboratory,	725 ft.
Latitude,	40° 6' 40"
Value of "g",	980.1141
Per cent difference in two results,	.001

1. Smithsonian Physical Tables, p. 104.

VIII. DISCUSSION

The swinging of a pendulum under reduced pressure furnishes a good opportunity to study the effect of the air on the period. The data show that the period is very appreciably decreased and that the apparent value of "g" is somewhat larger. At first thought it would seem that this effect is due to a decrease in the viscosity of the air, but Maxwell has deduced from the Kinetic Theory of Gases[1], that within wide limits the viscosity of a gas is independent of the pressure. This fact has been verified by numerous experiments. In 1660 Boyle first showed that the vibrations of a pendulum die away at the same rate, irrespective of the pressure. It was found, however, that for pressures below .01 mm., the viscosity decreased very rapidly with the pressure. It is evident then, that for the conditions of this experiment, the viscosity factor does not enter into consideration.

The reason for the decrease in the period is found in the buoyant effect of the air. At the reduced pressure the weight of the pendulum is increased by an amount equal to the weight of the air displaced at atmospheric pressure. However, with the increase in weight, it must be remembered that the mass of the pendulum remains constant. The increased pull on the pendulum makes it travel thru its path more rapidly, thus decreasing the period. The buoyant effect of the air acts in opposition to the force of gravity, hence, when the air is removed, the force of gravity shows a slight increase. These statements are suppor-

1. Poynting and Thomson, "Properties of Matter", p. 218.

ted by the experimental results obtained.

The accuracy of the final result was somewhat hindered because no provision was made for a very accurate measurement of the amplitude of vibration . The method used gives the period with reasonable accuracy out to the seventh decimal place, while the correction for arc appears in the fifth place. Since the amplitude could not be measured to an accuracy greater than one or two millimeters, a correction for the arc farther out than the fifth place is hardly justifiable. For this reason the period was carried only to the fifth place thruout. This however, gives the period correctly to six significant figures, and with the measurement of the length also to six figures, it is reasonable to believe that the final value for "g" is correct to six figures.

For purposes of comparison, the value of "g" is also computed from a formula given in the Smithsonian Physical Tables. The establishment of this formula will not be considered here. The absolute value of "g" at sea level and latitude 45° (980.600) is taken as the basis, so that by substituting the elevation and latitude for a given place the value of "g" is readily calculated. The elevation of the laboratory was obtained from a benchmark established by the United States Geological Survey, and the latitude was furnished by the astronomical department. The formula also depends on the ratio of the surface density to the mean density of the earth. This is a factor which is very likely subject to local variation, and which would account for the slight discrepancy in the two results.

[illegible]

[illegible]

[illegible]

[illegible]

It was found that the coincidences could be observed more accurately with the pendulum swinging thru a fairly large arc. The pendulum was usually started at an amplitude of about 2 cm. measured at a distance of 123.9 cm. from the point of support. With smaller amplitudes the velocity thru the middle point is not sufficient to produce a clear and distinct differentiation of the images just before and just after a coincidence. For this reason observations were not continued longer than two hours after starting the pendulum.

To obtain a closer measurement of "g" than that accomplished in this work, there are a few other points that might well be considered. The standard meter bar that was used in the measurement of the distance between the knife-edges was assumed to be correct at 0°C. For a verification of this, it should be standardized by comparison with the standard bar kept at the United States Bureau of Standards at Washington. Since the bar is of invar and not subject to atmospheric changes, it is not likely that any error in the bar would affect the measurement even in the fourth decimal place.

As the knife-edges are made of steel and the receiver of iron, magnetic effects are possible. With a magnetic needle, it was found that the receiver showed a slight magnetization,with the lower end a north pole, and the upper a south pole. The mass of the steel knife-edges is exceedingly small in comparison with that of the brass pendulum, so that, with the receiver very feebly magnetized, the effect on the period need not be considered.

It was found that the knife-edges made a slight im-

pression in the iron plate which served as the support. The effect of this is to increase the friction of suspension, which causes the vibrations of the pendulum to die away more rapidly. This would have no appreciable effect on the period of the pendulum.

The time required for the vibrations of the pendulum to die away at reduced pressure was not perceptibly different from that at atmospheric pressure. This shows that the viscosity of the air is not affected, and thus bears out Maxwell's deduction from the Kinetic Theory of Gases.

In conclusion, I wish to express my appreciation to Prof. Carman for his advice in carrying out the work, to Dr. Stebbins for furnishing the astronomical data, and to Mr. Hayes for his pains in the construction of the apparatus.